MAXIMAL VALUES OF THE CONDUCTIVE COMPONENT OF THE EXTERNAL HEAT TRANSFER IN MISCIBLE DISPERSED MEDIA

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On the basis of a two-zone heat-transfer model, a universal form of the dependence of the maximal magnitude of the conductive component of the external heat transfer on governing factors is set up.

The question of the magnitudes of the maximally achievable heat-transfer coefficients under specific conditions occurs often in the development of apparatus with dispersed media. These quantities govern the possibilities of the technique being created and they must be able to be computed sufficiently exactly. Quite important for disperse layers of different type is the clarification of universal heat-transfer regularities and the creation of simple and similar dependences for the computation of $\alpha_{\max}$ on their basis. Precisely such a program was first noted in the researches of Antonishin and colleagues [1, 2]. On the basis of a two-temperature model for the external heat transfer they had developed earlier [3], a simple universal equation was obtained that describes the dependence of the maximal coefficient of conductive heat transfer on governing factors in terms of the nonstationary heat transfer of "packets" of particles with the surface

$$\mathrm{Nu}_{\max} = (A^{-2} + \mathrm{Fo}_{\beta})^{-0.5}. \tag{1}$$

Selecting specific values of the constant interphasal heat transfer A and the exposure time of the "packet" at the heat-transfer surface β, the authors of [1, 2] succeeded in describing the quantity $\alpha_{\max}$ in different miscible dispersed media by using (1). The fruitfulness of such an approach is obvious. However, a certain ambiguity should be noted in the selection of the quantities A and β, which is associated with the selection of λ_{ef} and ε that enter into (1), as the authors themselves remark.

A two-zone heat-transfer model was proposed in [4, 5] to describe the external heat transfer in purged dispersed media. It takes account of the influence of the two most important factors, the zone of elevated porosity at the heat-transfer surface and the heat conductivity of the dispersed medium, on the heat transfer.

Let us use the representation underlying this model to establish a universal form of the dependence of $\alpha_{\max}$ on the governing parameters.

The system of stationary heat-transport equations in a one-dimensional approximation has the form

$$\frac{d}{dy}\left(\lambda_f \frac{dT_f}{dy}\right) - \frac{c_f \rho_f u (T_f - T_0)}{H\varepsilon} = 0, \quad 0 \leqslant y < l_0; \tag{2}$$

$$\lambda_s \frac{d^2 T_s}{dy^2} - \frac{c_f \rho_f u (T_s - T_0)}{H} = 0, \quad l_0 < y \leqslant l. \tag{3}$$

Taking account of the influence of the dependence of the gas heat-conduction coefficient on the temperature on the heat transport is related to the fact that the main temperature drop is concentrated in the gas film at the heat-transfer surface. Analysis of a system analogous to (2) and (3) was performed in [4] in application to heat transfer in a fluidized bed and showed that in the case of a sufficiently intensive particle mixing in the layer (the value of the ratio is $\lambda_s/\lambda_f \sim 10^4$-$10^5$) practically the whole thermal resistivity is concentrated in the gas film and the conductive component of the total heat-transfer coeffi-

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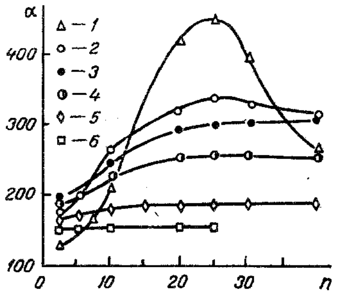

Fig. 1. Dependence of the heat-transfer coefficient of a rotating magnesite layer on the rate of activator rotation [7]: 1) d = 0.02-0.05 mm; 2) 0.06-0.10; 3) 0.10-0.16; 4) 0.20-0.315; 5) 0.40-0.63; b) 0.63-1.0. α, W/(m²·K); n, 1/sec.

cient reaches its maximal value. In this connection, we analyzed experimental data available in the literature on the conductive heat transfer in other dispersed media when intensive particle mixing is produced in vibrationally fluidized [6] and rotating [7-9] layers. Certain characteristic dependences are presented in Fig. 1. It is seen well that for definite magnitudes of the control parameters the coefficient α also reaches its maximal value, which is limited only by the thermal resistivity of the gas film within the framework of the two-zone model. This permits utilization of the two-zone heat transfer model representations developed earlier to set up a universal form of the dependence of the maximal heat-transfer coefficient and the effective thickness of the gas film on the governing factors in miscible dispersed media.

As was noted above, the thermal resistivity of the dispersed medium itself cannot be taken into account for the achievement of $\alpha_{\max}$. Therefore, it is sufficient to utilize just (2) in the system (2) and (3). Because of the small thickness of the gas film ($\ell_0 \sim (0.1\text{-}0.7)d$) and the moderate u the number $\mathrm{Pe} = c_f\rho_f u\ell_0^2/(H\varepsilon\langle\lambda_f\rangle) < 1$ and the term reflecting the influence of heat entrainment from the layer being filtered by the gas can be neglected in (2). For boundary conditions of the first kind, the problem of the external conductive heat transfer of a plane surface from a miscible dispersed medium has the form

$$\frac{d}{dy}\left(\lambda_f\frac{dT_f}{dy}\right)=0, \tag{4}$$

$$T_f(0)=T_w;\quad T_f(l_0)=T_s. \tag{5}$$

In the case of a linear dependence of λ_f on T_f: $\lambda_f = \lambda_f^0 + BT_f$, the solution of (4) and (5) can be written thus [10]:

$$T_f=-\frac{\lambda_f^0}{B}+\sqrt{\left(\frac{\lambda_f^0}{B}\right)^2+\frac{2}{B}\left[\frac{\lambda_f^0(T_s-T_w)+\frac{B}{2}(T_s^2-T_w^2)}{l_0}\,y+\lambda_f^0T_w+\frac{B}{2}T_w^2\right]}. \tag{6}$$

We obtain for $\alpha_{\max}$ from (6)

$$\alpha_{\max}=\frac{\lambda_f(T_w)\dfrac{dT_f(0)}{dy}}{T_s-T_w}=\frac{\lambda_f^0+B(T_s+T_w)/2}{l_0}=\frac{\langle\lambda_f\rangle}{l_0}. \tag{7}$$

The derivation of the magnitude of the governing temperature, equal to $(T_w + T_s)/2$, is in agreement with the results in [11] obtained on the basis of an analysis of data on the magnitudes of $\alpha_{\max}$ in high-temperature fluidized beds.

In conformity with (7), the problem of finding $\alpha_{\max}$ reduces to determining the effective thickness ℓ_0 of the gas film. Selecting the Archimedes number and the porosity of the dispersed layer as governing parameters, we can set

$$\frac{l_0}{d}=\varphi(\mathrm{Ar},\ \varepsilon)=C\,\mathrm{Ar}^{n_1}(1-\varepsilon)^{n_2} \tag{8}$$

on the basis of dimensional analysis. The presence of ε reflects the influence of broadening of the unfilterable miscible layer (fluidized, say, see [4]) on ℓ_0.

Taking (8) into account the expression (7) for $\alpha_{\max}$ takes the form

$$\alpha_{\max}=\frac{\langle\lambda_f\rangle}{d\varphi(\mathrm{Ar},\ \varepsilon)} \tag{9}$$

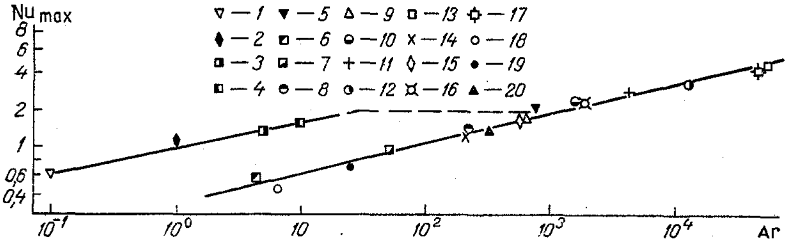

Fig. 2. Heat transfer of rotating dispersed layers at normal and reduced pressures: 1-5) data of [9] (lead glass d = 0.20 mm); 1) p = 13.3 Pa; 2) 133.3; 3) 666; 4) 1,330; 5) 101,300; 6-20) data of [7] (atmospheric pressure); 6-13) magnesite; 14-17) quartz sand; 18-20) corundum; 6) $d_{av} = 0.035$ mm; 7) 0.08; 8) 0.13; 9) 0.18; 10) 0.26; 11) 0.35; 12) 0.51; 13) 0.80; 14) 0.13; 15) 0.18; 16) 0.26; 17) 0.80; 18) 0.035; 19) 0.055; 20) 0.13.

TABLE 1. Effective Gas Film Thickness

Ar	Fluidized bed (11) $l_0/d = 1{,}16\mathrm{Ar}^{-0{,}20}$	Rotating layer (12) $l_0/d = 2{,}78\mathrm{Ar}^{-0{,}22}$	Vibrationally fluidized bed (13) $l_0/d = 0{,}83\mathrm{Ar}^{-0{,}16}$	Layer in a rotating furnace (14) $l_0/a = 0{,}95\mathrm{Ar}^{-0{,}19}$
10^1	0,73	1,67	0,57	—
10^2	0,46	1,01	0,40	0,40
10^3	0,29	0,61	0,27	0,26
10^4	0,18	0,37	0,19	0,16
10^5	0,12	0,22	0,13	0,11

or in dimensionless form

$$\mathrm{Nu}_{\max} = \frac{1}{l_0/d} = \frac{1}{\varphi(\mathrm{Ar},\ \varepsilon)} = \frac{1}{C}\,\mathrm{Ar}^{-n_1}(1-\varepsilon)^{-n_2}. \tag{10}$$

Relationships of the type (10) are utilized traditionally to generalize test data on the maximal values of the conductive heat-transfer component in fluidized beds. Let us mention just two, the Varygin and Martyushin formulas [12]:

$$\mathrm{Nu}_{\max} = 0{,}86\,\mathrm{Ar}^{0.20}\,{}^{*} \quad (3\cdot 10^1 < \mathrm{Ar} < 2\cdot 10^5), \tag{11}$$

and the Botterill and Denloye formula [13] obtained from generalizing test data at elevated fluidizing gas pressures

$$\mathrm{Nu}_{\max} = 0{,}843\,\mathrm{Ar}^{0{,}15} \quad (10^3 < \mathrm{Ar} < 2\cdot 10^6). \tag{11a}$$

Let us use the expression obtained for $\mathrm{Nu}_{\max}$ in the form (10) to generalize test data on the maximal magnitudes of the heat-transfer coefficient in other miscible dispersed media.

1. ROTATING LAYER

Presented in Fig. 2 are results of the generalization we made for the quantities $\alpha_{\max}$ obtained in [7, 9] at room temperature. For the tests at atmospheric pressure the generalized dependence has the form

$$\mathrm{Nu}_{\max} = 0.36\,\mathrm{Ar}^{0{,}22} \quad (5\cdot 10^{-1} < \mathrm{Ar} < 5\cdot 10^4). \tag{12}$$

The test data [9, p. 204] in the 13.3-1330 Pa pressure range are described by the formula

$$\mathrm{Nu} = 1.0\,\mathrm{Ar}^{0{,}21} \quad (10^{-1} < \mathrm{Ar} < 10^1). \tag{12a}$$

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*Under optimal fluidization conditions when $\alpha = \alpha_{\max}$, the ε_{opt} is a function of Ar and the dependence $\varphi(\mathrm{Ar},\ \varepsilon_{opt})$ is transformed into $\psi(\mathrm{Ar})$.

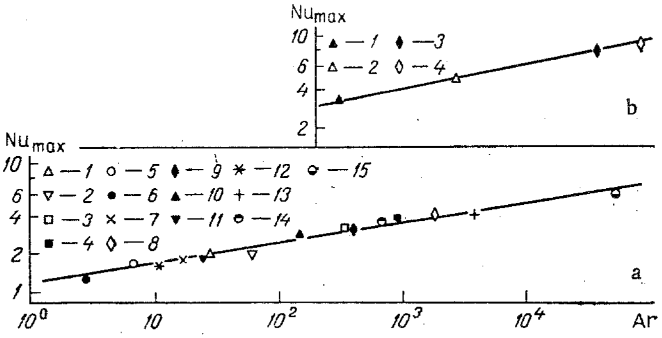

Fig. 3. Heat transfer in a vibrationally fluidized bed (a) and in a rotating granular charge layer (b): a) test data [6]: 1-5) $\langle T_f \rangle$ = 120°C, 6-15) 320°C; 1) d = 0.079 mm (carborundum), 2) d = 0.114 (sand), 3) d = 0.201 (sand), 4) d = 0.282 (sand), 5) d- 0.046 (corundum), 6) d = 0.046 (corundum), 7) d = 0.084 (corundum), 8) d = 0.471 (sand), 9) d = 0.282 (sand), 10) d = 0.201 (sand), 11) d = 0.114 (sand), 12) d = 0.079 (carborundum), 13) d = 0.400 (nickel), 14) d = 0.288 (corundum), 15) d = 1.0 (iron shot); b) Lemberg test data taken from [8]: 1) d = 0.16 mm, 2) 0.32, 3) 0.79, 4) d = 1.04.

2. VIBRATIONALLY FLUIDIZED BED

The generalization of test data from [6] is shown in Fig. 3a. The following equation is obtained

$$\mathrm{Nu}_{\max} = 1.2\,\mathrm{Ar}^{0.16} \quad (2 < \mathrm{Ar} < 5 \cdot 10^4), \tag{13}$$

in which the fluidizing gas characteristics are taken as the temperature $(T_s + T_w)/2$.*

3. DISPERSED LAYER IN ROTATING FURNACES

Shown in Fig. 3b are results of processing the Lemberg test data presented in [8]

$$\mathrm{Nu}_{\max} = 1.05\,\mathrm{Ar}^{0.19} \quad (3 \cdot 10^2 < \mathrm{Ar} < 8.7 \cdot 10^4). \tag{14}$$

Presented in the table are values of the effective gas film thicknesses calculated by means of (11)-(14) with the relationship (10) taken into account. As is seen, the values of ℓ_0/d for the majority of dispersed systems are in sufficiently good mutual agreement and are within the limits 1.67-0.11. It is interesting to note that $\ell_0/d \approx 0.10$ in fixed layers (see [5], for instance). This value of ℓ_0/d is apparently the limit for miscible layers. It permits estimation of the maximal values of the conductive heat-transfer coefficients achievable in miscible dispersed layers $\mathrm{Nu}_{\lim} = 1/(\ell_0/d)_{\lim} \approx 10$. The physical situation assuring this limit value of Nu is that the porosity of a dispersed layer close to the heat-transfer surface is close to the porosity of a fixed layer; hence a sufficiently intensive mixing of the particles is assured that permits neglecting the thermal resistivity of the granular layer. As follows from the table, vibrationally fluidized layers and layers in rotating furnaces for Ar ≥ 10^5 approach most closely to such a case. It is easy to see that the mentioned situation assuring the maximal value of the conductive stationary heat-transfer coefficient is modeled well in the nonstationary heat-transfer process of a fixed unpurged layer for very small times when the value of the heat-transfer coefficient is also determined completely only by the magnitude of the contact thermal resistivity (the gas film

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*An attempt was made to generalize the data of [6] by computing the quantities ρ_f and η_f in Ar at the temperature of the dispersed layer core (T_s = 40°C). In such a treatment the test points in the $\ln \mathrm{Nu}_{\max}$–$\ln \mathrm{Ar}$ coordinates were "stratified" and grouped around two equidistant lines corresponding to T_w = 200 and 600°C.

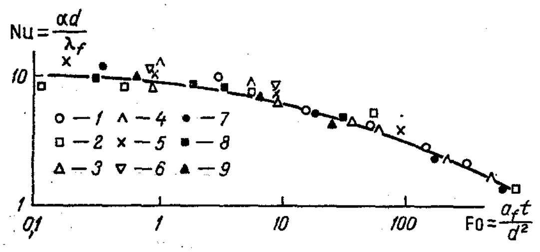

Fig. 4. Nonstationary heat transfer in fixed layers [14]: 1-3) d = 0.39, 2.07, and 5.05 mm (glass balls—air); 4-6) d = 0.39, 0.93, 3.05 mm (glass balls—CO_2), 7-9) d = 0.39, 0.93, 2.07 mm (glass balls—freon-12).

resistivity is R_f ℓ_0/λ_f). The value of $\ell_0 = 0.1d$ for a fixed layer also yields $\lim\limits_{Fo\to 0} Nu = d/R_f\lambda_f \approx 10$ (see Fig. 4).

In conclusion, let us emphasize that the regularities of external heat transfer in the manifold of miscible dispersed media considered and the conditions for their realization are described by very similar dependences (11)-(14), which indicates the commonality of the heat-transfer mechanism in such systems. The relationships (11)-(14) contain parameters that are easily determinable and convenient for utilization in engineering practice.

NOTATION

$A = \sqrt{6(1-\varepsilon)Nu^*}$, interphasal heat-transfer constant; c, specific heat; d, particle diameter; H, dispersed layer height; ℓ_0, efficient gas film thickness; ℓ, dispersed layer width; T, temperature; T_0, gas input temperature; $\langle T_f\rangle = (T_w + T_s)/2$; u, rate of gas filtration; y, coordinate normal to the heat-transfer surface; α^*, interphasal heat-transfer coefficient; α, conductive component of the external heat-transfer coefficient; β, exposure time of the particle "packet" at the heat-transfer surface; ε, porosity; λ, heat conductivity; $\lambda_f{}^0$, gas heat conductivity for $T_f = 0°C$; $\langle\lambda_f\rangle$, value of λ_f for $\langle T_f\rangle = (T_s + T_w)/2$; η, viscosity; ρ, density; $Ar = gd^3\rho_f(\rho_s - \rho_f)/\eta_f{}^2$; $Fo_\beta = \lambda_{ef}\beta/c_f\rho_f(1-\varepsilon)d^2$, dimensionless time of "packet" exposure; $Nu^* = \alpha^* d/\lambda_{ef}$; $Nu = \alpha d/\langle\lambda_f\rangle$. Subscripts: f, gas; s, solid particles; w, heat-transfer surface; ef, effective.

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